

## Editorial: Special issue on stability of non-linear dynamic structures and systems

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This special issue is dedicated to the stability of non-linear dynamic structures and systems. The majority of the papers in this issue, i.e. [1–8], discuss macro-scale structures and systems, which typically can be found in mechanical, civil, automotive, and aerospace engineering. In addition, in [9], a micro-electromechanical switch and, in [10], a tapping mode micro-beam for atomic force microscopy (AFM) are discussed.

In literature, the concept stability is used in several contexts. Therefore, we first introduce the contexts, which apply here. The primary context of stability in this special issue is that stability of dynamic structures and systems may be lost due to the occurrence of non-linear resonances caused by periodic excitation [1–7]. Undesirable non-linear resonances, which may be closely related to the occurrence of bifurcations, may lead to large displacements, large rotations, and/or large deformations. As a result, malfunctioning, dynamic buckling, fatigue, or total collapse of the structure or system may occur. Dynamic buckling due to step loading using a finite element based reduction method is considered in [8] and dynamic snap-through

of a micro-beam due to ramp and step loading is investigated in [9]. In the latter case, two-directional snap-through is desired because the micro-beam serves as a switch. Finally, in [10], white noise added to harmonic excitation brings an AFM micro-beam from a non-contact dynamic state to a desired dynamic tapping contact state.

In this special issue, the concept stability is also frequently encountered with respect to periodic solutions caused by periodic or pure harmonic excitation [1–7]. The local stability of periodic solutions is evaluated using Floquet theory, which can be extended for systems of Filippov-type by introducing saltation matrices, as reviewed in [6]. The global stability of a periodic solution can be determined by investigating its domain of attraction; see, e.g. [4]. In this special issue, investigation of the local/global stability of periodic solutions is subservient to the above sketched primary context of stability.

Several papers in this issue pay attention to multi-physics modeling. The mechanical model is coupled to thermal loads in [1], to an electrodynamic shaker in [3], to the fluid domain in [4], to fringing electrostatic fields in [9], and, for their macro-structure, to magnetic forces in [10]. We now briefly summarize the content of the papers included in this special issue.

Alijani et al. [1] investigate geometrically non-linear vibrations of rectangular plates with Functionally Graded Materials (FGM) in thermal environments via a multi-modal energy approach. Both non-linear first-order shear deformation theory and von Karman

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theory are used to model simply supported FGM plates with movable edges. Using Lagrange's equations of motion, the energy functional is reduced to a system of infinite non-linear ordinary differential equations with quadratic and cubic non-linearities. It is revealed that, in order to obtain the accurate natural frequency in thermal environments, an analysis based on the full non-linear model is unavoidable since the plate loses its original flat configuration due to thermal loads. Both the effects of temperature variations and the volume fraction exponent are discussed and it is illustrated that thermally deformed FGM plates have stronger hardening behavior; on the other hand, the effect of the volume fraction exponent is not significant, but modal interactions may rise in thermally deformed FGM plates that could not be seen in their undeformed isotropic counterparts. Bifurcation diagrams of Poincaré maps and maximum Lyapunov exponents are obtained in order to detect and classify bifurcations and complex non-linear dynamics.

Demeio et al. [2] investigate the primary non-linear resonance response of a one-dimensional continuous system, which can be regarded as a model for semi-infinite cables resting on an elastic substrate reacting in compression only, and subjected to a constant distributed load and to a small harmonic displacement applied to the finite boundary. By introducing a straightforward small amplitude expansion, first the frequencies of the oscillations of the system about the static solution are determined at all orders. It is found that, at each order, there exists a critical (cut-off) frequency, above which the solution behaves as a traveling wave toward infinity, while it decays exponentially below it. Then the resonance response of the system is examined, when an external harmonic excitation is applied at the finite boundary. Using Multiple-Time-Scale analysis, the third-order bending of the resonance curves, whose hardening or softening behavior depends upon the frequency of the chosen primary resonance, is determined.

Fey et al. [3] investigate non-linear resonances in a coupled shaker-beam-top mass system both numerically and experimentally. The imperfect, vertical beam carries the top mass and is axially excited by the shaker at its base. The weight of the top mass is below the beam's static buckling load. A semi-analytical model is derived for the coupled system. In this model, Taylor-series approximations are used for the inextensibility constraint and the curvature of the beam. In the

model with a single beam mode, parametric and direct resonances are found, which affect the dynamic stability of the structure. The model with two beam modes not only shows an additional second harmonic resonance, but also reveals some extra small resonances in the low-frequency range, some of which can be identified as combination resonances. The experimental steady-state response is obtained by performing a (stepped) frequency sweep-up and sweep-down with respect to the harmonic input voltage of the amplifier-shaker combination. A good correspondence between the numerical and experimental steady-state responses is obtained.

Using Donnell's non-linear shallow shell equations in terms of the displacements and the potential flow theory, Silva et al. [4] present a qualitatively accurate low dimensional model to study the non-linear dynamic behavior and stability of a fluid-filled cylindrical shell under lateral pressure and axial loading. First, the reduced order model is derived taking into account the influence of the driven and companion modes. For this, a modal solution is obtained, which satisfies exactly the in-plane equilibrium equations and all boundary, continuity, and symmetry conditions. Finally, the equation of motion in the transversal direction is discretized by the Galerkin method. The quality of the proposed model is corroborated by studying the convergence of frequency–amplitude relations, resonance curves, bifurcation diagrams, and time responses. The global response of the system is investigated in order to quantify the degree of safety of the shell in the presence of external perturbations through the use of bifurcation diagrams and basins of attraction.

Stoykov and Ribeiro [5] investigate the geometrically non-linear periodic vibrations of beams with rectangular cross section under harmonic forces using a p-version finite element method. The beams vibrate in space, hence they experience longitudinal, torsional, and non-planar bending deformations. The model is based on Timoshenko's theory for bending and assumes that, under torsion, the cross section rotates as a rigid body and is free to warp in the longitudinal direction, as in Saint-Venant's theory. The theory employed is valid for moderate rotations and displacements, and physical phenomena like internal resonances and change of the stability of the solutions can be investigated. Green's non-linear strain tensor and Hooke's law are considered and isotropic and elastic

beams are investigated. The equations of motion are derived by the principle of virtual work, are then converted into a non-linear algebraic form employing the harmonic balance method, and finally solved by the arc-length continuation method. The variation of the amplitude of vibration in space due to changes in the excitation frequency is determined and is presented in the form of response curves.

In the first part of their study, Theodosiou et al. [6] present the basic steps of a methodology, leading to the periodic steady state response (and its stability) of periodically excited mechanical models of Filippov-type with contact and dry friction. The analytical part is complemented by a suitable continuation procedure, enabling evaluation of complete branches of periodic motions. In the second part of the study, numerical results are obtained for selected examples. The first two examples are single degree-of-freedom oscillators, one with contact and one with friction. The last example is a more involved and challenging model, related to the function of an engine valve and characterized by large numerical stiffness.

Yabuno et al. [7] investigate the non-linear normal modes of a horizontally supported Jeffcott rotor. In contrast with a vertically supported rotor, there are localized and non-localized non-linear normal modes because the linear natural frequencies in the horizontal and vertical directions are slightly different due to both gravity and the non-linearity of the restoring force. Reflecting such non-linear normal modes, the frequency response curves are characterized in the primary resonance. In the case where the eccentricity is small, i.e., the response amplitude is small, the whirling motion is localized in the horizontal or vertical direction in the resonance. On the other hand, when the eccentricity is large, two kinds of whirling motion, which are localized in the vertical direction and non-localized in any direction, co-exist simultaneously in a region of rotational speed. Experiments are conducted, and the theoretically predicted non-linear responses based on localized and non-localized non-linear normal modes are observed.

Rahman et al. [8] present a finite element formulation of a reduction method for dynamic buckling analysis of imperfection-sensitive shell structures. The reduction method makes use of a perturbation approach, initially developed for static buckling and later extended to dynamic buckling analysis. The implementation of a single-mode dynamic buckling analysis in a

general purpose finite element code is described. The effectiveness of the approach is illustrated by application to the dynamic buckling of composite cylindrical shells under axial and radial step loads. Results of the reduction method are compared with results available in the literature. The results are also compared with full model finite element explicit dynamic analysis, and a reasonable agreement is obtained.

Krylov et al. [9] investigate the feasibility of two-directional switching of an initially curved or pre-buckled electrostatically actuated micro-beams using a single electrode fabricated from the same structural layer. The distributed electrostatic force, which is engendered by the asymmetry of the fringing fields in the deformed state, acts in the direction opposite to the deflection of the beam. A reduced order model was built using the Galerkin decomposition with linear undamped modes of a straight beam as base functions and verified using the results of the numerical solution of the differential equation. Static stability analysis reveals that the presence of the restoring electrostatic force may result in the suppression of the snap-through instability as well as in the appearance of additional stable configurations associated with higher buckling modes of the beam that are not observed in “mechanically” loaded structures. It is shown that two-directional switching of a pre-buckled beam between two stable configurations cannot be achieved using quasi-static loading. Furthermore, it is shown that switching is associated with the dynamic snap-through mechanism and possible within certain interval of actuation voltages. Using a single degree-of-freedom (lumped) model, estimation voltage boundaries are obtained. Theoretical results illustrate the feasibility of the suggested operational principle as an efficient mechanism in the arena of non-volatile mechanical memory devices.

Chakraborty and Balachandran [10] study the influence of noise on the dynamics of base-excited elastic cantilever structures at the macro-scale and micro-scale. The macro-scale system is a base excited cantilever structure whose tip experiences long-range attractive and short-range repulsive forces similar in form to tip interaction forces in tapping mode AFM. The macro-scale system is used to study non-linear phenomena and apply the associated findings to the chosen AFM application. The repulsive forces are modeled as Derjaguin–Muller–Toporov contact forces in both the macro-scale and micro-scale system, and the attractive forces are modeled as van der

Waals attractive forces in the micro-scale system and as magnetic attractive forces in the macro-scale system. A single-mode model is used to numerically study the response for a combined deterministic and random base excitation. In the macro-scale system, it is experimentally and numerically found that the addition of white Gaussian noise to a harmonic base excitation facilitates contact between the tip and the sample, when there was previously no contact with only the harmonic input, and results in a response that is nominally close to a period-doubled orbit associated with near-grazing impacts between the tip and the sample. The effects of Gaussian white noise are numerically studied for a tapping mode AFM application, and it is shown that contact between the tip and the sample can be realized by adding noise of an appropriate level to a harmonic excitation.

As exemplified by the above brief summaries of the content of this special issue, investigation of the stability of non-linear dynamic structures and systems plays an important role in a wide range of (multi-physics) engineering applications, both on macro-scale and on micro-scale. We hope that this special issue showing different approaches in modeling, numerical analysis, theoretical analysis, and experimental analysis will form an inspiration to readers to make further advancements in this research field. Moreover, we hope that it promotes cooperation between scientists and engineers working in different disciplines.

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